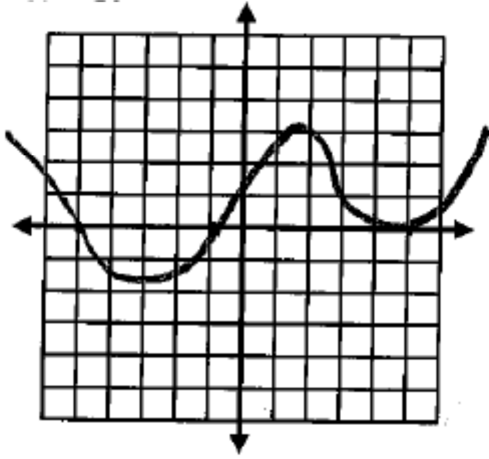


Review - Derivatives

AP Calculus BC

Name _____

1. The figure below shows the graph of f' , the derivative of the function f , for $-7 \leq x \leq 7$. The graph has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.



- Find all values of x , for $-7 < x < 7$, at which f attains a relative minimum.
- Find all values of x , for $-7 < x < 7$, at which f attains a relative maximum.
- Find all values of x , for $-7 < x < 7$, at which $f''(x) < 0$.
- At what value of x , for $-7 \leq x \leq 7$, does f attain its absolute maximum? Justify your answer.

2. Consider the curve given by $xy^2 - x^3y = 6$.

a) Find $\frac{dy}{dx}$.

- b) Find all points on the curve whose x -coordinate is 1, and write an equation for the tangent line of each of these points.

- c) Find the x -coordinate of each point on the curve where the tangent line is vertical.

3. Let f be the function given by $f(x) = 3 \cos x$. 2 points on the curve are: $P(0, 3)$ and $Q(\frac{\pi}{2}, 0)$.

a) Write an equation for the line passing through points P and Q .

b) Write an equation for the line tangent to the graph of f at point Q .

c) Find the x -coordinate of the point on the graph of f , between points P and Q , at which the line tangent to the graph of f is parallel to the line through P and Q .

MULTIPLE CHOICE: No calculator

1. If $y = xy + x^2 + 1$, then when $x = -1$, $\frac{dy}{dx} =$

- a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) -1 d) -2 e) does not exist

2. If $y = 2x - 8$, what is the minimum value of the product xy ?

- a) -16 b) -8 c) -4 d) 0 e) 2

3. A particle moves along the x -axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6 , at what time t during the interval $0 \leq t \leq 4$ is the particle farthest to the right?

- a) 0 b) 1 c) 2 d) 3 e) 4

4. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

- a) $\frac{3x-3}{\sqrt{2x-3}}$ b) $\frac{x}{\sqrt{2x-3}}$ c) $\frac{1}{\sqrt{2x-3}}$ d) $\frac{-x+3}{\sqrt{2x-3}}$ e) $\frac{5x-6}{2\sqrt{2x-3}}$

5. if $f(x) = x^3 + x + \frac{1}{x}$, then $f'(-1) =$

- a) 3 b) 1 c) -1 d) -3 e) -5

6. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

- a) $x < 0$ b) $x > 0$ c) $x < -2$ or $x > -2/3$ d) $x < 2/3$ or $x > 2$ e) $2/3 < x < 2$

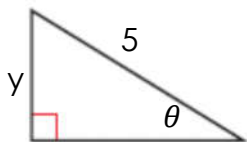
7. $\frac{d}{dx} \cos^2(x^3) =$

- a) $6x^2 \sin(x^3) \cos(x^3)$ b) $6x^2 \cos(x^3)$ c) $\sin^2(x^3)$ d) $-6x^2 \sin(x^3) \cos(x^3)$ e) $-2 \sin(x^3) \cos(x^3)$

8. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find than approximation to a zero of f , that approximation is

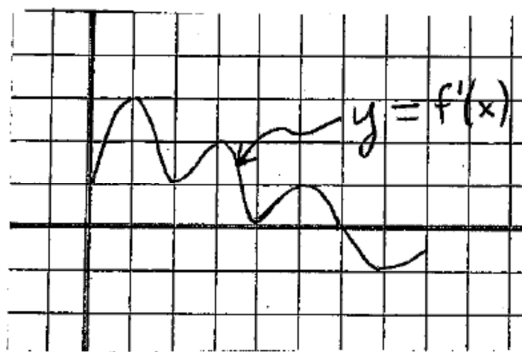
- a) 0.4 b) 0.5 c) 2.6 d) 3.4 e) 5.5

9. In the triangle shown below, if θ increases at a constant rate of 3 radians per minute, at what rate is y increasing in units per minute when $y = 3$ units?



- a) 3 b) $15/4$ c) 4 d) 9 e) 12

For #10 – 12, use the graph below. The function f is defined on $[0, 8]$. The graph below is the graph of the derivative of f .



10. The point $(3, 5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3, 5)$ is:

- a) $y = 2$ b) $y = 5$ c) $y - 5 = 2(x - 3)$ d) $y + 5 = 2(x - 3)$ e) $y + 5 = 2(x + 3)$

11. How many points of inflection does the graph of f have?

- a) 2 b) 3 c) 4 d) 5 e) 6

12. At what value of x does the absolute minimum occur?

- a) 0 b) 2 c) 4 d) 6 e) 8

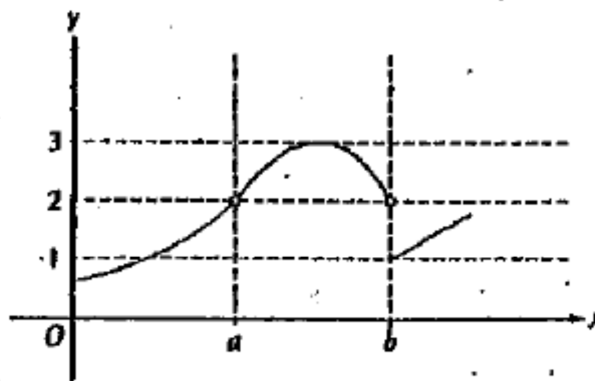
13. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \frac{\sin^2 x}{2}$ II. $F(x) = \frac{\cos^2 x}{4}$ III. $F(x) = \frac{-\cos(2x)}{4}$

- a) I only b) II only c) III only d) I and III only e) II and III only

14. The graph of the function f is shown in the figure below. Which of the following statements about f is true?

- a) $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow b} f(x)$
- b) $\lim_{x \rightarrow a} f(x) = 2$
- c) $\lim_{x \rightarrow b} f(x) = 2$
- d) $\lim_{x \rightarrow b} f(x) = 1$
- e) $\lim_{x \rightarrow a} f(x)$ does not exist



MULTIPLE CHOICE: With calculator

15. At time $t \geq 0$, the acceleration of a particle moving on the x -axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2 . For what value of t will the velocity of the particle be zero?

- a) 1.02 b) 1.48 c) 1.85 d) 2.81 e) 3.14

16. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

- a) 57.60 b) 57.88 c) 59.20 d) 60.00 e) 67.40

17. The graph of the function $y = x^3 + 6x^2 + 7x - 2 \cos x$ changes concavity at $x = ?$

- a) -1.58 b) -1.63 c) -.67 d) -1.89 e) -2.33

18. $\lim_{x \rightarrow \infty} \frac{10^8 x^5 + 10^6 x^4 + 10^4 x^2}{10^9 x^6 + 10^7 x^5 + 10^5 x^3} =$

- a) 0 b) 1 c) -1 d) 1/10 e) -1/10

19. $\lim_{x \rightarrow -3} \frac{x^2 + 3x}{\sqrt{x^2 + 6x + 9}}$ is

- a) -3 b) -1 c) 1 d) 3 e) nonexistent

20. $\lim_{x \rightarrow 3} \frac{(3-x)^2}{(x-3)} =$

- a) 0 b) -2 c) 1 d) -1 e) nonexistent

21. If $\lim_{x \rightarrow 3} \frac{g(3) - g(x)}{3 - x} = -0.628$, then at a point $x = 3$, the graph of $g(x)$

- a) is decreasing b) is increasing c) is concave up
d) is concave down e) attains a relative minimum

22. If f is a function such that $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = 0$, which of the following must be true?

- a) $\lim_{x \rightarrow a} f(x)$ does not exist b) $f(a)$ does not exist c) $f'(a) = 0$
d) $f(a) = 0$ e) $f(x)$ is continuous at $x = 0$

23. The $\lim_{h \rightarrow 0} \frac{|x+h| - |x|}{h}$ at $x = 3$ is

- a) 0 b) 1 c) 3 d) -1 e) nonexistent

24. If f is a function which is everywhere increasing and concave upwards, which statement is true about f^{-1} , the inverse of f ?
- (A) f^{-1} is not a function.
(B) f^{-1} is increasing and concave upwards.
(C) f^{-1} is increasing and concave downwards.
(D) f^{-1} is decreasing and concave upwards.
(E) f^{-1} is decreasing and concave downwards.
25. If f is a continuous odd function and the $\lim_{x \rightarrow -\infty} f(x) = -3$, which of the following statements must be true?
- I. $\lim_{x \rightarrow +\infty} f(x) = 3$
II. There are no vertical asymptotes.
III. The lines $y = 3$ and $y = -3$ are horizontal asymptotes.
- (A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III
26. If the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches?
- (A) 800π (D) 40π
(B) 800 (E) 80π
(C) 3200π

Answers:

1. a) $x = -1$ b) $x = -5$ c) $(-7, -3) \cup (2, 5)$

d) $x = 7$. F increases on the entire interval from $x = -1$ to $x = 7$.

2. a) $\frac{3x^2y - y^2}{2xy - x^3}$ b) $(1, 3) y = 3$ and $(1, -2) y = 2x - 4$ c) $x = \sqrt[5]{24}$

3. a) $y = -\frac{6}{\pi}x + 3$ b) $y = -3x + \frac{3\pi}{2}$ c) $x = \sin^{-1}(2/\pi) \approx .690$ (use calculator)

Multiple Choice:

1. B
2. B
3. B
4. A
5. D
6. E
7. D
8. C
9. E
10. C
11. E
12. A
13. D
14. B
15. B
16. A
17. D
18. A
19. E
20. A
21. A
22. C
23. B
24. C
25. E
26. A